INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 40, Northern Spring 2019 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Several positive integers that sum to 20, are written in sequence. Neither a number in the sequence, nor a sum of two or more adjacent numbers is equal to 3. Is it possible that such a sequence contains more than 10 numbers? (3 points)

2. There are $2n + 1$ coins placed around a circle. Initially, all the coins are heads up. Proceeding clockwise $2n + 1$ flips are performed: one coin is flipped, the next coin is skipped, the next coin is flipped, the next two coins are skipped, the next coin is flipped, the next three coins are skipped, the next coin is flipped and so on, until, finally $2n$ coins are skipped and the next coin is flipped. Prove that after the $2n + 1$ flips described above, exactly one coin is heads down. (4 points)

3. The product of two positive integers m and n is divisible by their sum. Prove that $m + n \leq n^2$. (4 points)

4. Isosceles triangles with a fixed apex angle α (at the vertex opposite to the base) are inscribed in rectangle $ABCD$ so that their apex vertices lie on the side BC and the vertices of their bases lie on the sides AB and CD respectively. Prove that the midpoints of the bases of all possible locations for those triangles coincide. (5 points)

5. A magician and his assistant present the following trick. Twelve empty closed boxes are placed in a row. Then, the magician leaves the stage, and a random person from the audience is selected to put two coins into two boxes of their choice, one coin in each box, in front of the magician's assistant, i.e. the assistant knows which boxes contain coins. Then, the magician returns and his assistant is allowed to open one box that does not contain a coin. After that the magician must choose four boxes to be opened simultaneously. The goal of the magician is to open both boxes with coins. Construct a scheme by which the magician and his assistant can perform the trick successfully every time. (5 points)